EE 232 Lightwave Devices
Lecture 7: Absorption and Gain Coefficient, Bernard-Duraffourg Inversion Condition, Lineshape

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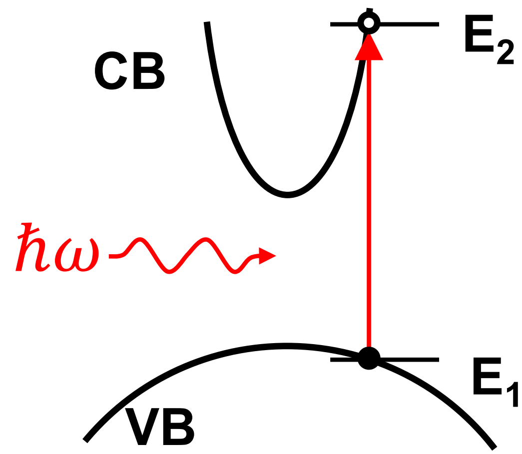
Absorption Coefficient

When CB and VB are partially filled:

Absorption Condition: VB is occupied, CB is empty

Absorption probability $= f_V(E_1)\left(1 - f_C(E_2)\right)$

$$R_{12}(\hbar\omega) = \frac{2}{V}\sum_k \left[\frac{2\pi}{\hbar}\left|H'_{ba}\right|^2 \delta(E_b - E_a - \hbar\omega)\right] f_V\left(1 - f_C\right)$$

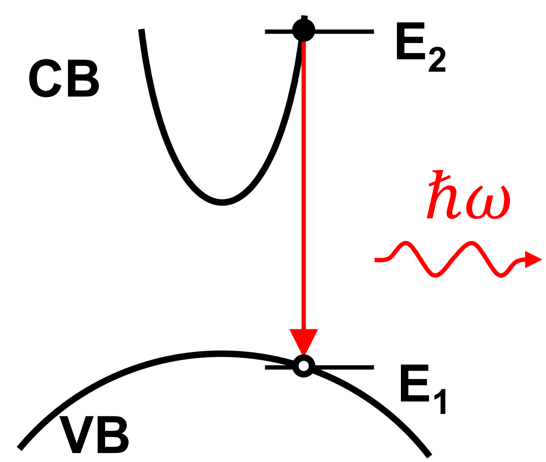


$$R_{12} \propto f_V\left(1 - f_C\right)$$

Emission Condition: CB is occupied, VB is empty

Emission probability $= f_C(E_2)\left(1 - f_V(E_1)\right)$

$$R_{21}(\hbar\omega) = \frac{2}{V}\sum_k \left[\frac{2\pi}{\hbar}\left|H'_{ba}\right|^2 \delta(E_b - E_a - \hbar\omega)\right] f_C\left(1 - f_V\right)$$



$$R_{21} \propto f_C\left(1 - f_V\right)$$

Absorption Coefficient

Net absorption rate:

$$R(\hbar\omega) = \frac{2}{V}\sum_k \left[\frac{2\pi}{\hbar}\left|H_{ba}^{'}\right|^2 \delta(E_b - E_a - \hbar\omega)\right]\left[f_V\left(1-f_C\right) - f_C\left(1-f_V\right)\right]$$

$$= \frac{2\pi}{\hbar}\left|H_{ba}^{'}\right|^2 \int \frac{2d\vec{k}}{(2\pi)^3}\delta(E_g + \frac{\hbar^2 k^2}{2m_r^*} - \hbar\omega)\left[f_V(k) - f_C(k)\right]$$

$$= \frac{2\pi}{\hbar}\left|H_{ba}^{'}\right|^2 \int \frac{2d\vec{k}}{(2\pi)^3}\delta(E_g + \frac{\hbar^2 k^2}{2m_r^*} - \hbar\omega)\left[f_V(k) - f_C(k)\right]$$

$$= \frac{2\pi}{\hbar}\left|H_{ba}^{'}\right|^2 \rho_r(\hbar\omega - E_g)\left[f_V\left(-\left(\hbar\omega - E_g\right)\frac{m_r^*}{m_h^*}\right) - f_C\left(E_g + \left(\hbar\omega - E_g\right)\frac{m_r^*}{m_e^*}\right)\right]$$

$$\boxed{\alpha(\hbar\omega) = C_0\left|\hat{e}\cdot\vec{P}_{cv}\right|^2 \rho_r(\hbar\omega - E_g)\left[-f_g(\hbar\omega - E_g)\right]}$$

Fermi Inversion Factor :

$$\boxed{f_g(\hbar\omega - E_g) = f_C\left(E_g + \left(\hbar\omega - E_g\right)\frac{m_r^*}{m_e^*}\right) - f_V\left(-\left(\hbar\omega - E_g\right)\frac{m_r^*}{m_h^*}\right)}$$

Fermi Inversion Factor

$$f_g(\hbar\omega - E_g) = f_C(E_2) - f_V(E_1)$$

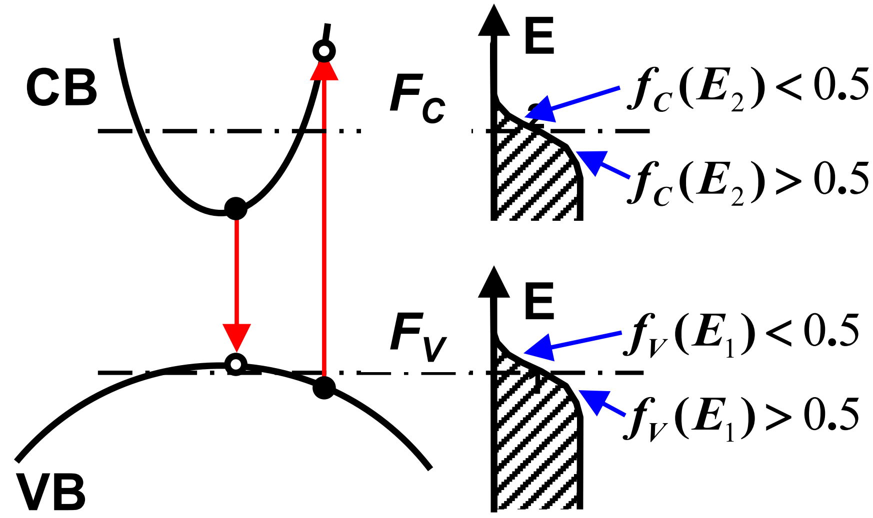
$$\begin{cases} E_2 = E_g + (\hbar\omega - E_g)\dfrac{m_r^*}{m_e^*} \\ E_1 = -(\hbar\omega - E_g)\dfrac{m_r^*}{m_h^*} \end{cases}$$

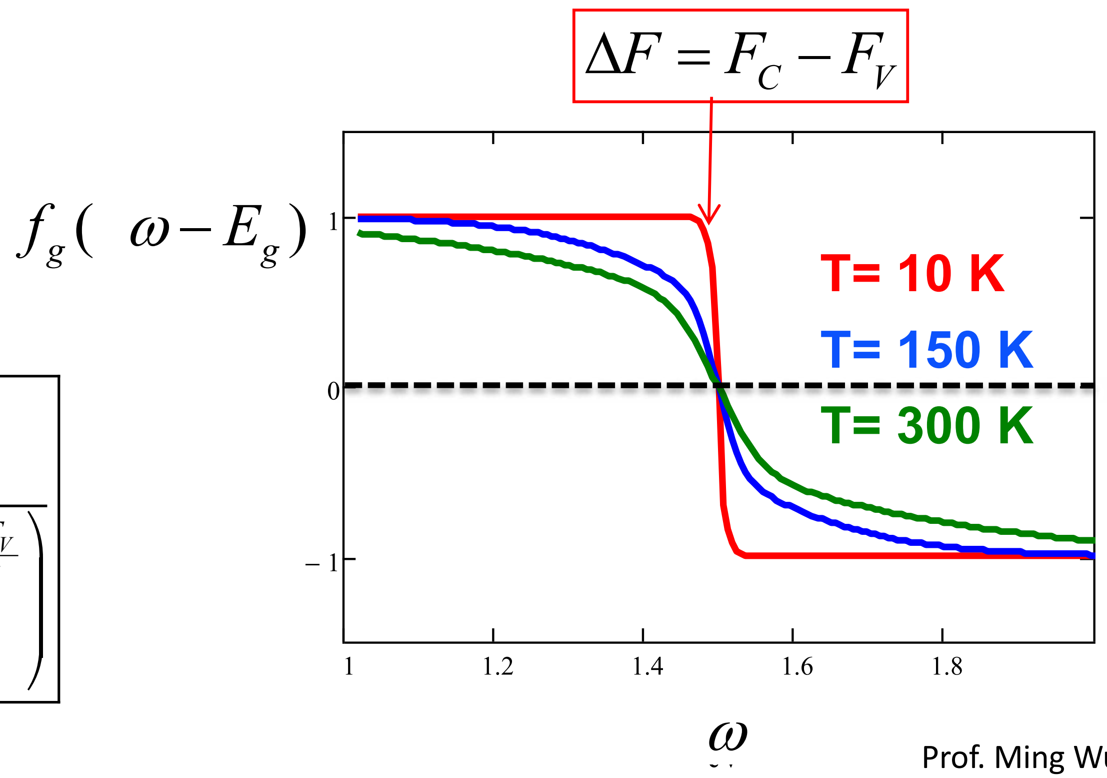
And $\quad k_1 = k_2$

$$f_C(E_2) = \frac{1}{1 + e^{\frac{E_2 - F_C}{k_B T}}}$$

$$f_V(E_1) = \frac{1}{1 + e^{\frac{E_1 - F_V}{k_B T}}}$$

$$f_g(\hbar\omega - E_g) = \frac{e^{\frac{E_1 - F_V}{k_B T}} - e^{\frac{E_2 - F_C}{k_B T}}}{\left(1 + e^{\frac{E_2 - F_C}{k_B T}}\right)\left(1 + e^{\frac{E_1 - F_V}{k_B T}}\right)}$$





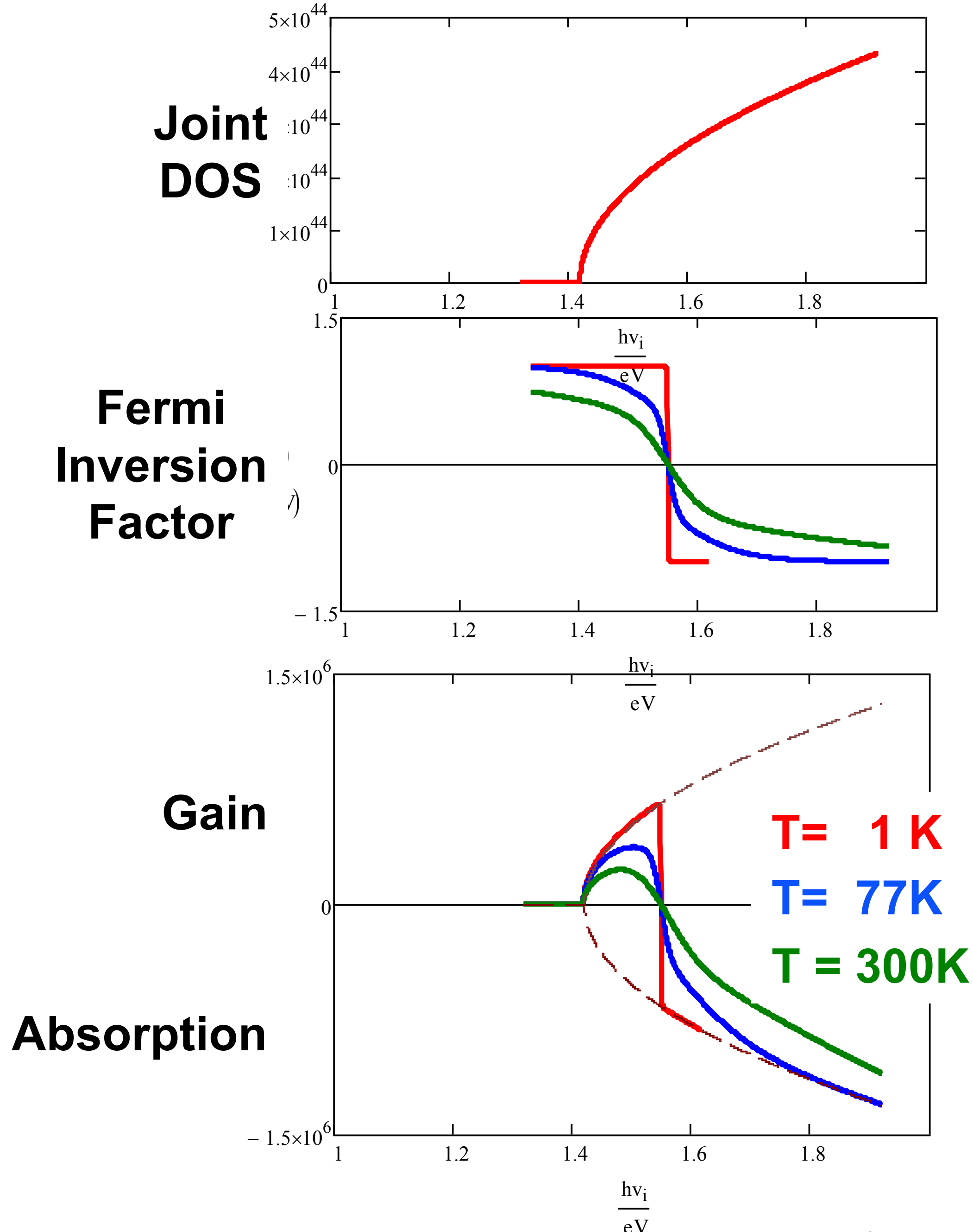
Optical Gain Coefficient

$$\alpha_0(\hbar\omega) = C_0 \left| \hat{e} \cdot \vec{P}_{cv} \right|^2 \rho_r(\hbar\omega - E_g)$$

$$\boxed{\alpha(\hbar\omega) = \alpha_0(\hbar\omega)\left[-f_g(\hbar\omega - E_g)\right]}$$

$$g(\hbar\omega) = -\alpha(\hbar\omega)$$

$$\boxed{g(\hbar\omega) = \alpha_0(\hbar\omega) f_g(\hbar\omega - E_g)}$$



Optical Gain Coefficient versus Bias

$$g(\hbar\omega)=\alpha_0(\hbar\omega)f_g(\hbar\omega-E_g)$$

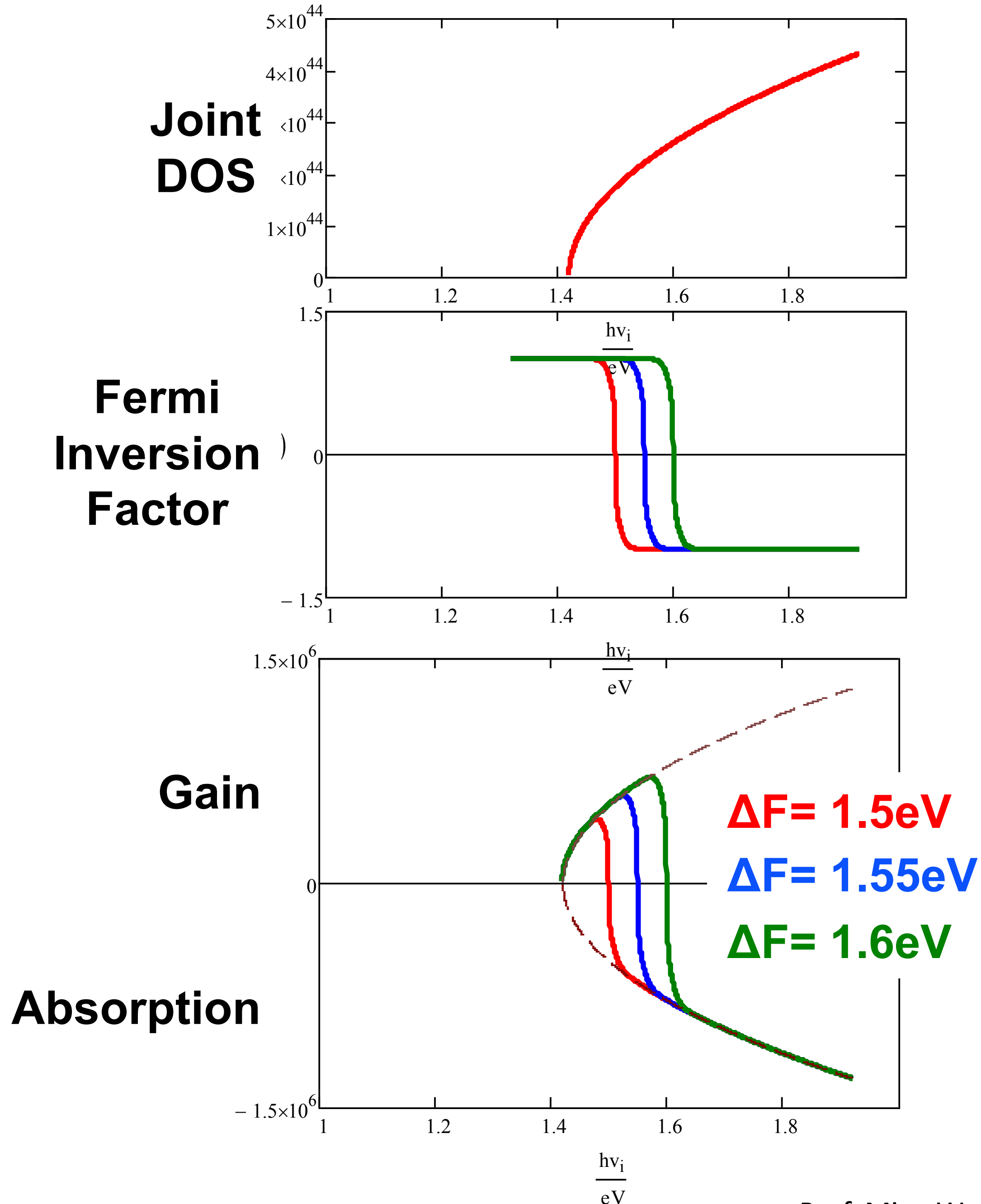
Bernard-Duraffourg

Inversion Condition

$$\boxed{\Delta F = F_C - F_V > E_g}$$

Spectral Range of Gain

$$\boxed{E_g < \hbar\omega < \Delta F = F_C - F_V}$$



Lineshape Broadening

$$g(\hbar\omega) = C_0 \left|\hat{e}\cdot\vec{P}_{cv}\right|^2 \int dE \rho_r(E)\delta(E = \hbar\omega - E_g) f_g(E)$$

Lineshape broadening due to intraband scattering of electrons

→ Replace delta function by a Lorentzian lineshape function:

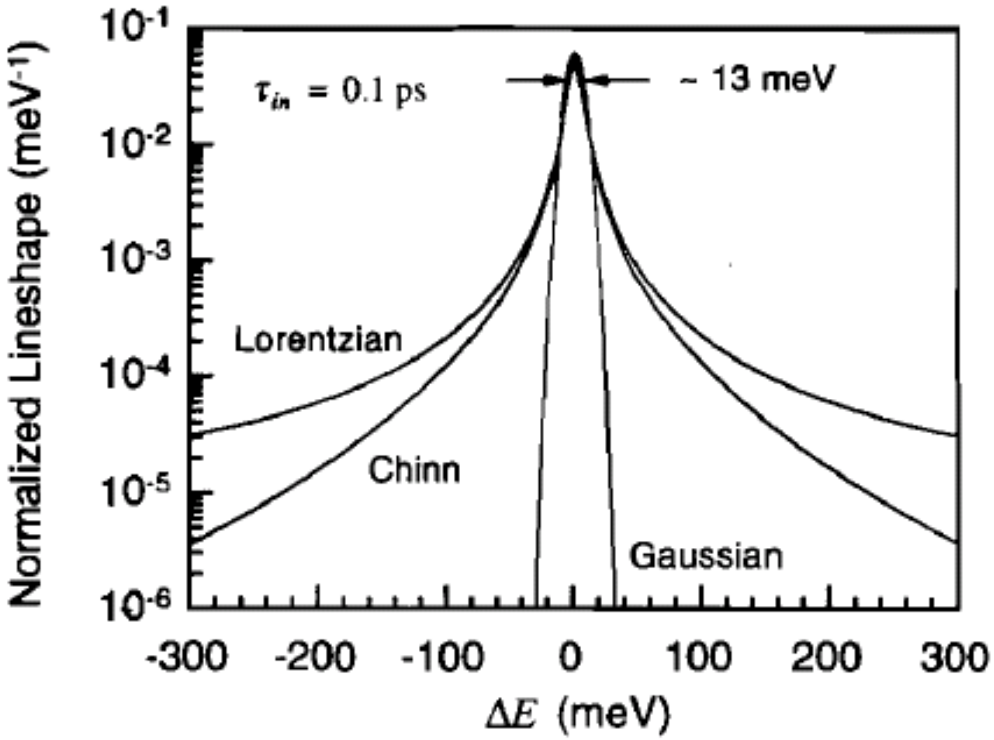
$$\delta(E = \hbar\omega - E_g) \to L\left(E - (\hbar\omega - E_g)\right) = \frac{1}{\pi}\frac{\hbar/\tau_{in}}{\left(E - (\hbar\omega - E_g)\right)^2 + \left(\hbar/\tau_{in}\right)^2}$$

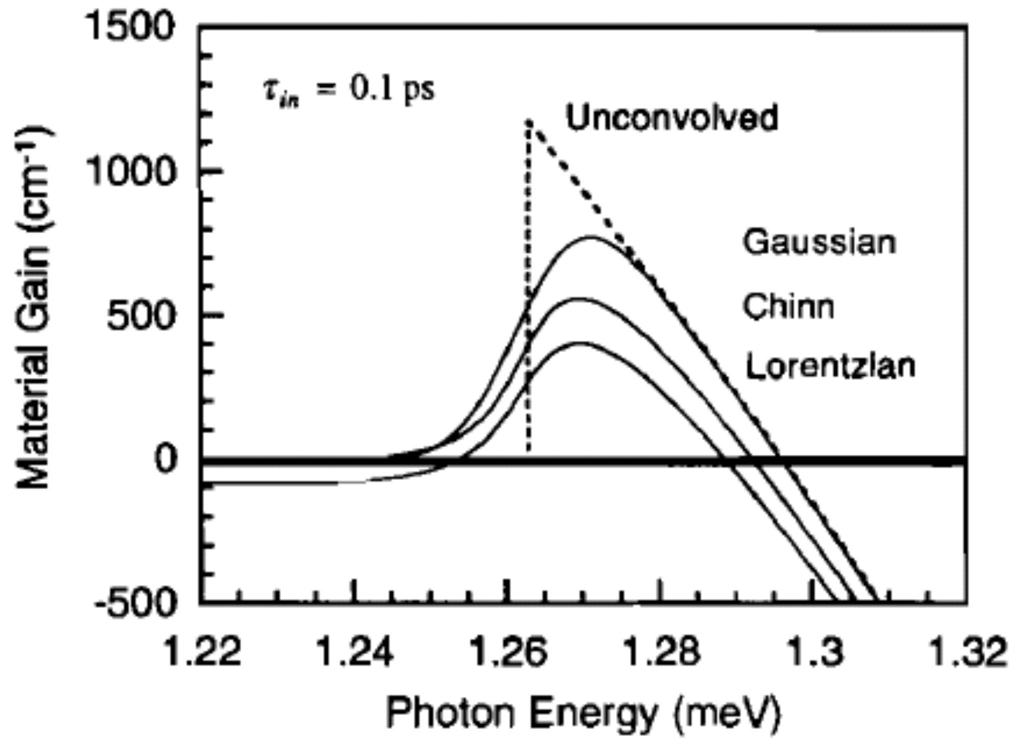
$$\text{FWHM} = \Gamma = \frac{2\hbar}{\tau_{in}}$$

τ_{in} : intraband relaxation time $\sim 0.1 ps$

$$g(\hbar\omega) = C_0 \left|\hat{e}\cdot\vec{P}_{cv}\right|^2 \int dE \rho_r(E)\left[\frac{1}{\pi}\frac{\hbar/\tau_{in}}{\left(E - (\hbar\omega - E_g)\right)^2 + \left(\hbar/\tau_{in}\right)^2}\right] f_g(E)$$

Lineshape Functions and Gain Spectra





Source: Coldren & Corzine, p.133